

p. 500 #13

$$r = -2x$$

Find a formula for the truncation error if we use $P_6(x)$ to approximate $\frac{1}{1+2x}$ on $(-.5, .5)$.

$$P_6(x) = 1 - 2x + 4x^2 - 8x^3 + 16x^4 - 32x^5 + 64x^6$$

error bound formula $|-128x^7| \rightarrow |-128(.5)^7|$

p. 500 20

a. If $\cos(x)$ is replaced by $1 - \frac{x^2}{2}$ and $|x| < .5$, what estimate can be made of the error? $-.5 < x < .5$

next term = $\frac{x^4}{4!} \rightarrow \frac{.5^4}{4!}$

$\cos x$ $\cos(0)$
 $1 - x^2$ $1 - 0^2$

b. Does $1 - \frac{x^2}{2}$ tend to be too large or too small.

too small b/c $\frac{x^2}{2}$ is neg
and $\cos x$ alternates

p. 500 #22

The approximation $\sqrt{1+x} \approx 1 + \frac{x}{2}$ is used when x is small. Estimate the error when

$|x| < .1$
 $-.1 < x < .1$

$f''(-.1)$

$f''(0) = -\frac{1}{4}$ $f''(.1)$

$$\left| \frac{-\frac{1}{4}(.1)^{3/2}}{2!} \right|^2$$

$f(x) = (1+x)^{1/2}$
 $f'(x) = \frac{1}{2}(1+x)^{-1/2}$
 $f''(x) = -\frac{1}{4}(1+x)^{-3/2}$
 $f'''(x) = \frac{-1}{4(1+x)^{3/2}}$

$$f(x) = (x-2)^{-1}$$

$$f'(x) = -(x-2)^{-2}$$

$$f''(x) = 2(x-2)^{-3}$$

$$f'''(x) = -6(x-2)^{-4}$$

$$f(x) = \ln(x-2) \quad f(3) = 0$$

$$f'(x) = \frac{1}{x-2} = 1$$

$$P_3(3.5) = .401$$

$$\ln|1.5| = .405$$

$$|P_3(x) - \ln(1.5)| =$$

p. 527 #60

Let $f(x) = \frac{1}{x-2}$ at $x = 3$.

a. Write the first 4 terms and the general term of the Taylor Series generated by $f(x)$ at $x = 3$.

$$f(3) = 1$$

$$f'(3) = -1$$

$$f''(3) = 2$$

$$f'''(3) = -6$$

$$P_3(x) = 1 - (x-3) + \frac{2(x-3)^2}{2} - \frac{6(x-3)^3}{3!}$$

$$P_3(x) = 1 - (x-3) + (x-3)^2 - (x-3)^3$$

$$\text{General Term } (-1)^n (x-3)^n$$

b. Use the result in part (a) to find the fourth order polynomial and the general term of the series generated by $\ln|x-2|$ at $x = 3$.

$$1(x-3) - \frac{1}{2}(x-3)^2 + \frac{1}{3}(x-3)^3 - \frac{1}{4}(x-3)^4 = 2n|x-2|$$

gen: $\frac{(-1)^n (x-3)^{n+1}}{n+1}$

$$\frac{1}{2} - \frac{1}{2}\left(\frac{1}{2}\right)^2 = .375$$

error $\frac{1}{4}(3.5-3)^4$

$$\frac{1}{3}(.5)^3 = .0416 \quad .015625$$

c. Use the series in part (b) to compute a number that differs from $\ln(1.5)$ by less than 0.05. Justify your answer.

$x = 3.5 \rightarrow \text{alt}$

$x = .5$

Actual error $\ln\left(\frac{3}{2}\right) = .375??$

use 2 terms

$$\frac{1}{5}(3.5-3)^5 \rightarrow \frac{1}{5}(.5)^5$$

error bound .00625

83. The Taylor Series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n}$. Let f be

the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $.3 \leq x \leq 1.7$ is

- (A) .030 (B) .039 (C) .145 (D) .153 (E) .529

$$P(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$$

$$|f(3) - P_3(3)| = .145$$

$$|f(1.7) - P_3(1.7)| = .039$$